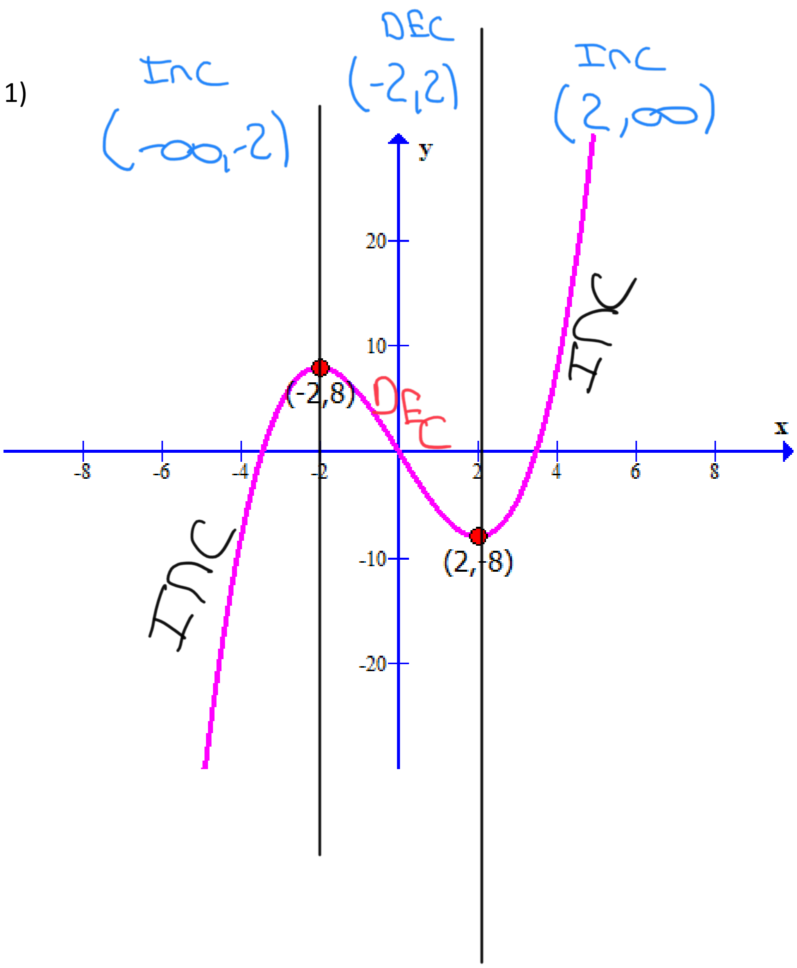
Section 3.1 Increasing and Decreasing functions and Relative Maxima and Minima
(Minimum Homework: 1 – 11 odds 13, 17, 21, 25)

1)



a) interval(s) where the graph is increasing. $(-\infty, -2) \cup (2, \infty)$

b) interval(s) where the graph is decreasing. $(-2,2)$

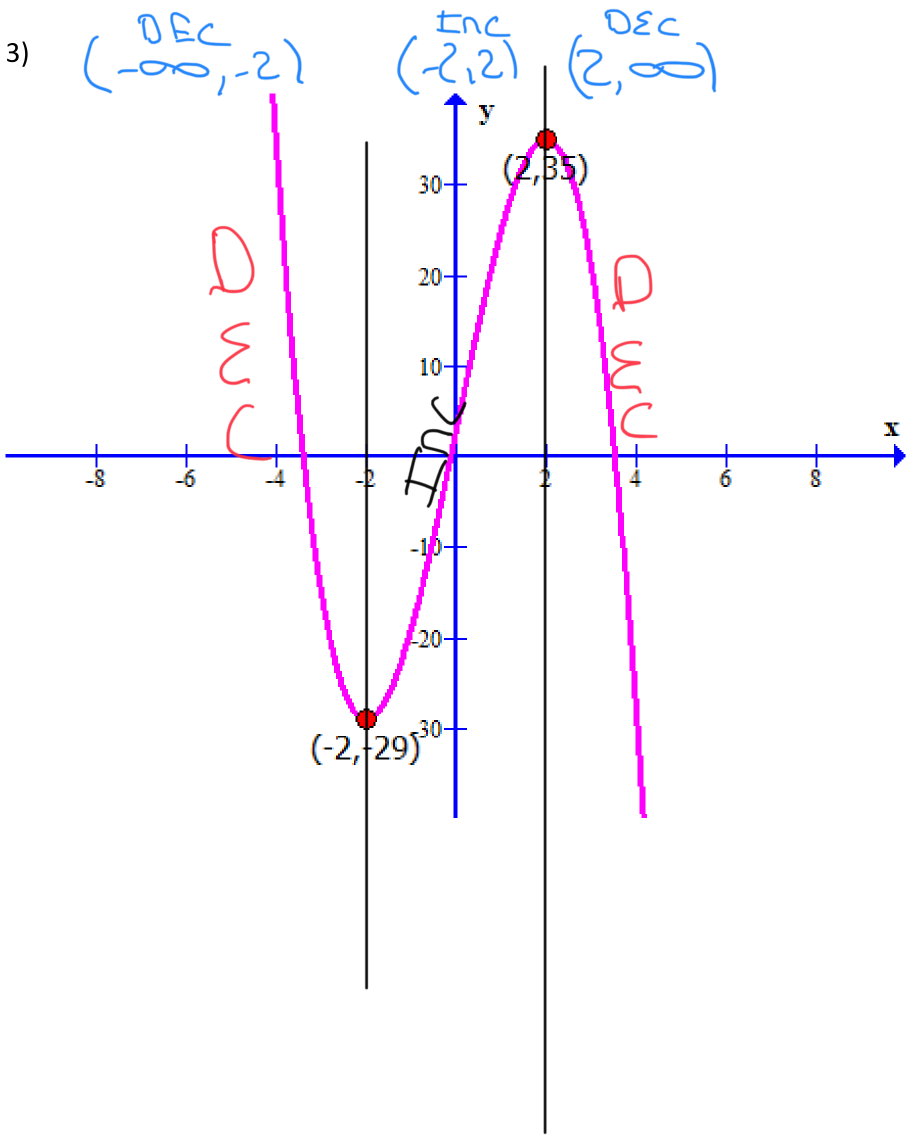
c) the coordinates of relative maximum point if any $(-2,8)$

d) the relative maximum value $y = 8 \; which \; occurs \; when \; x = \; -2$

e) the coordinates of the relative minimum point if any $(2, -8)$

f) the relative minimum value $y = \; -8 \; which \; occurs \; when \; x = 2$

3)



a) interval(s) where the graph is increasing. $(-2,2)$

b) interval(s) where the graph is decreasing. $(-\infty,-2) \cup (2,\infty)$

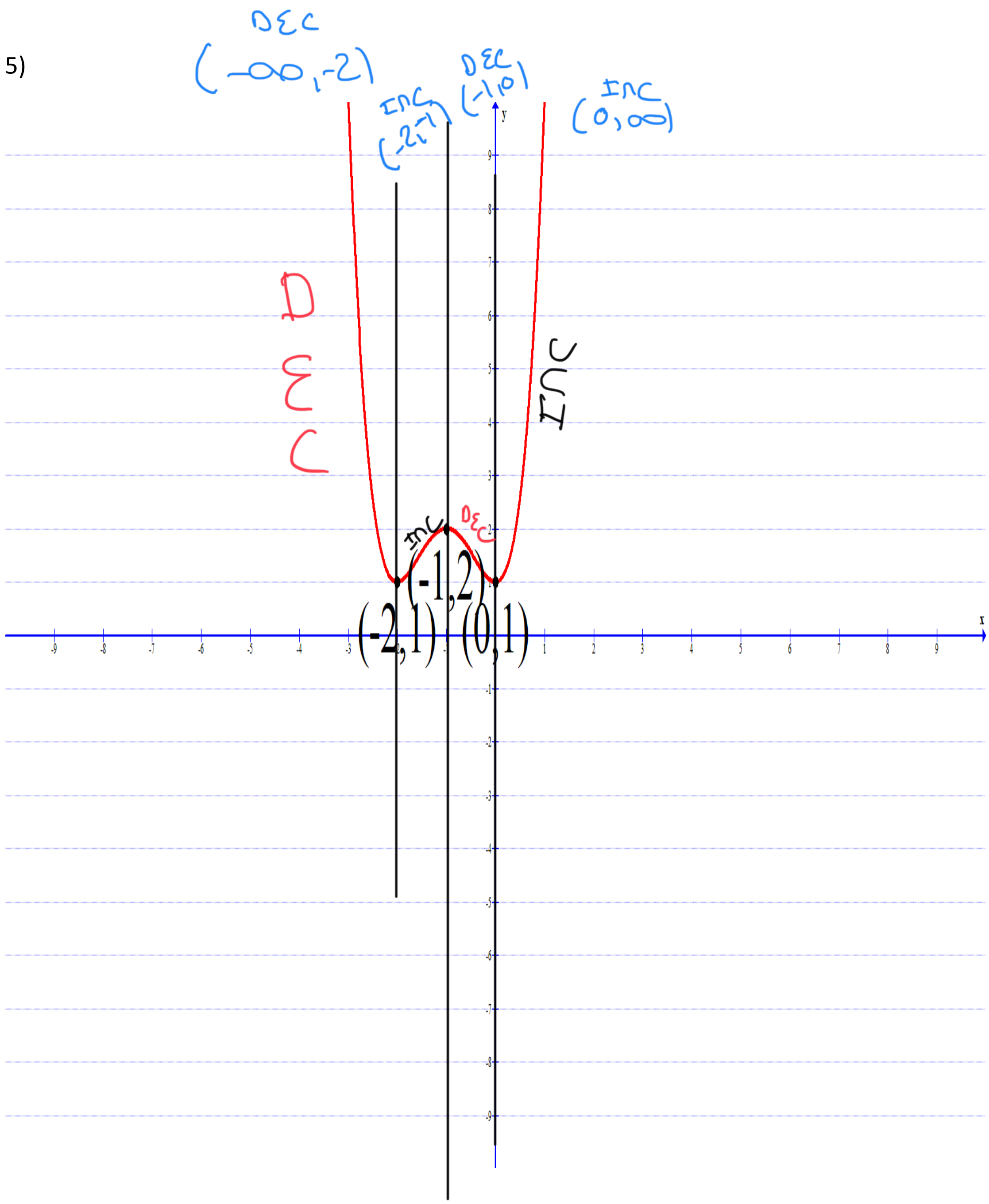
c) the coordinates of relative maximum point if any $(2,35)$

d) the relative maximum value $y = 35 \text{ which occurs when } x = 2$

e) the coordinates of the relative minimum point if any $(-2,-29)$

f) the relative minimum value $y = -29 \text{ which occurs when } x = -2$

5)



a) interval(s) where the graph is increasing. $(-2,1) \cup (0, \infty)$

b) interval(s) where the graph is decreasing. $(-\infty, -2) \cup (-1,0)$

c) the coordinates of relative maximum point if any $(-1,2)$

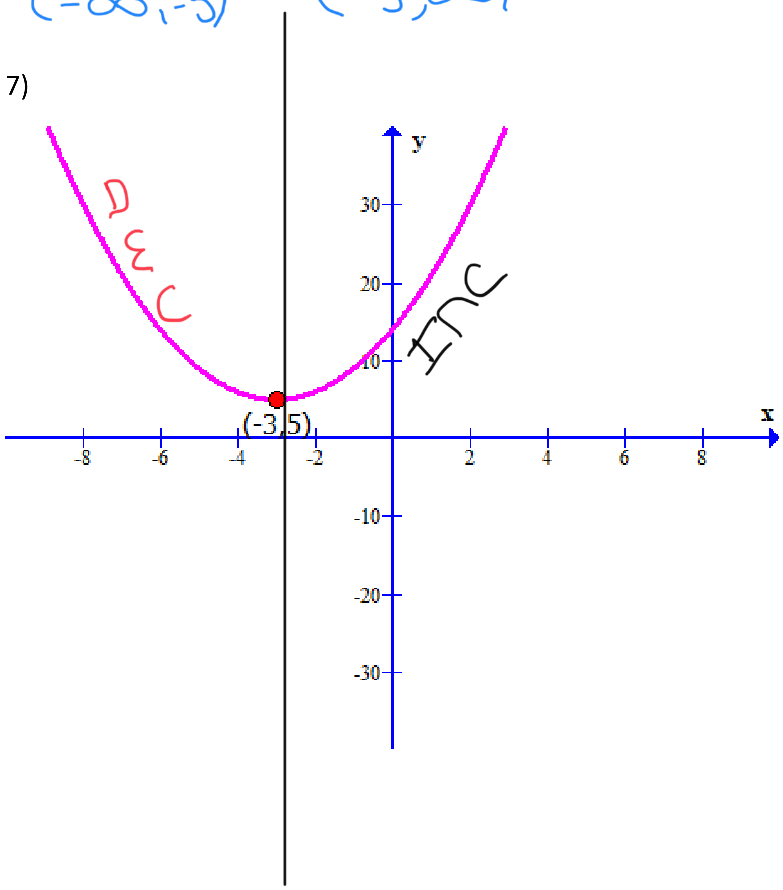
d) the relative maximum value $y = 2 \; which \; occurs \; when \; x = -1$

e) the coordinates of the relative minimum point if any $(-2,1) \; and \; (0,1)$

f) the relative minimum value $y = 1 \; when \; x = \; -2 \,, 0$

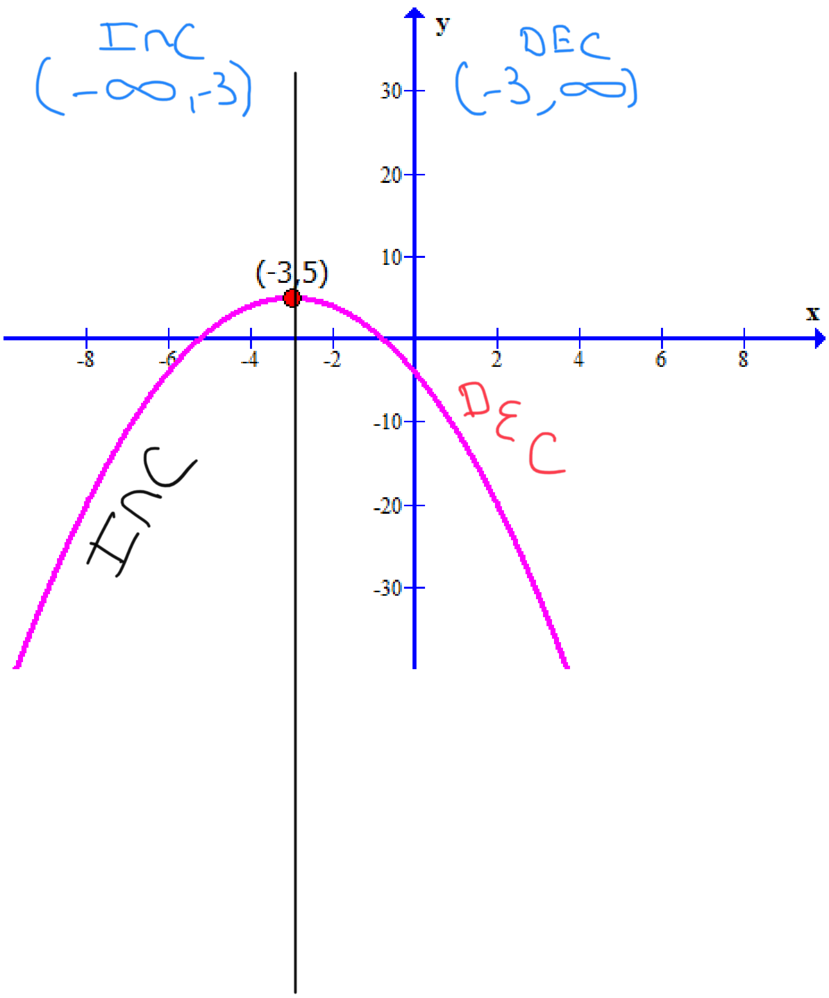
DEC $(-\infty,-3)$ Inc $(-3,\infty)$

7)



a) interval(s) where the graph is increasing. $(-3,\infty)$
b) interval(s) where the graph is decreasing. $(-\infty,-3)$
c) the coordinates of relative maximum point if any $none$
d) the relative maximum value $none$
e) the coordinates of the relative minimum point if any $(-3,5)$
f) the relative minimum value $y = 5\ which\ occurs\ when\ x = -3$

9)



a) interval(s) where the graph is increasing. $(-\infty, -3)$

b) interval(s) where the graph is decreasing. $(-3, \infty)$

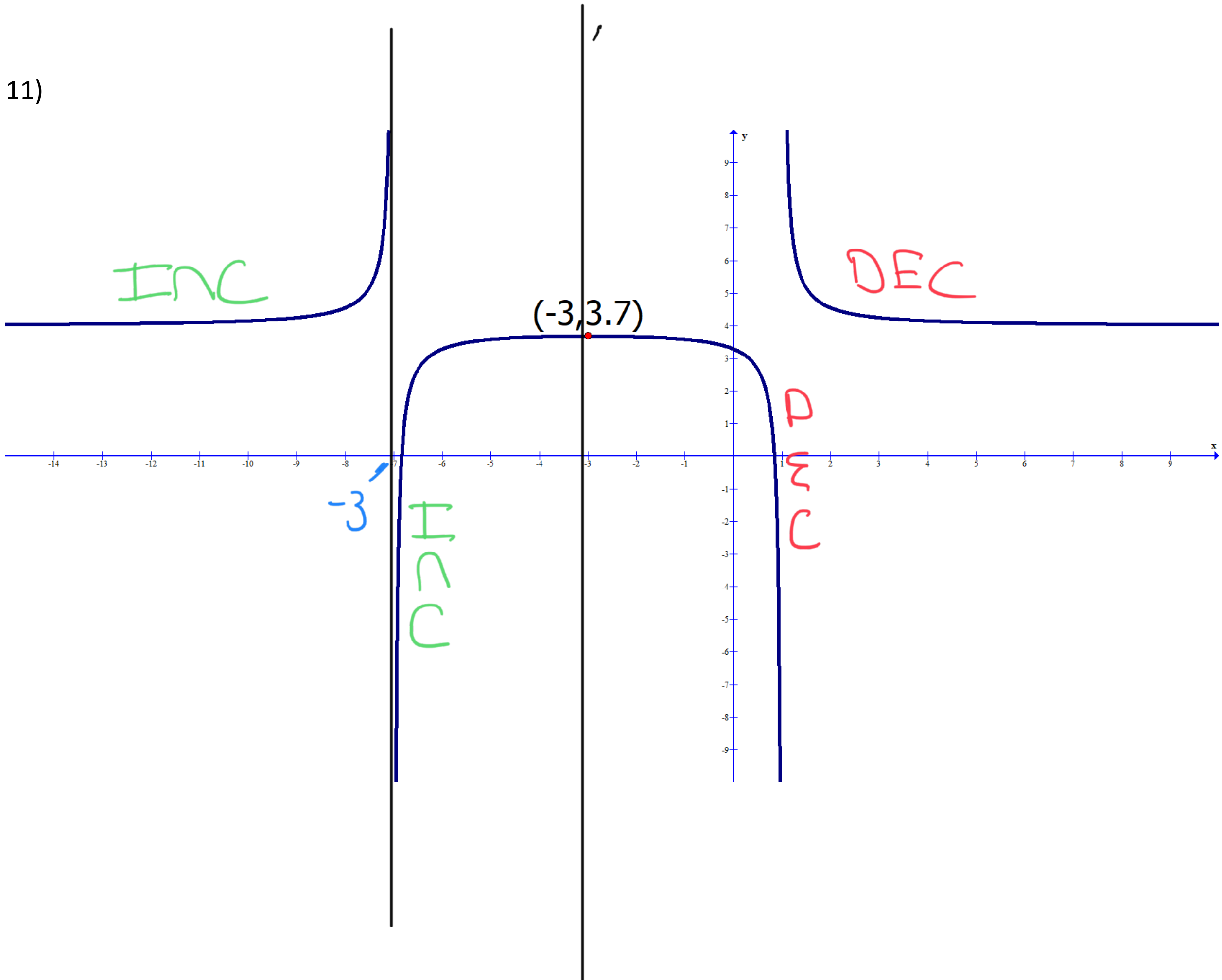
c) the coordinates of relative maximum point if any $(-3,5)$

d) the relative maximum value $y = 5 \; which \; occurs \; when \; x = -3$

e) the coordinates of the relative minimum point if any $none$

f) the relative minimum value $none$

11)



a) interval(s) where the graph is increasing. $(-\infty,-7)\cup(-7,-3)$
b) interval(s) where the graph is decreasing. $(-3,1)\cup(1,\infty)$
c) the coordinates of relative maximum point if any $(-3,3.7)$
d) the relative maximum value $y=3.7\ which\ occurs\ when\ x=\ -3$
e) the coordinates of the relative minimum point if any $none$
f) the relative minimum value $none$

(Minimum Homework: 1 – 11 odds 13, 17, 21, 25)

#13 – 26: For each function find the following:

a) $f'(x) = 2x - 6$

13) $f(x) = x^2 - 6x + 3$

b) $2x - 6 = 0$

$2x = 6$

$x = 3$

c, d, e, f

min (no max)

$-\infty$ $(-\infty, 3)$ 3 $(3, \infty)$ ∞

$x = 0$

$f'(0) = 2(0) - 6 = -6$

negative

- Decreasing

$x = 4$

$f'(4) = 2(4) - 6 = 2$

positive

- Increasing

no max

y-coord min $f(3) = (3)^2 - 6(3) + 3 = -6$

Min $(3, -6)$

a) f'(x) $f'(x) = 2x - 6$

b) the critical numbers $x = 3$

c) interval(s) where the graph is increasing. $(3, \infty)$

d) interval(s) where the graph is decreasing. $(-\infty, 3)$

e) the coordinates of relative maximum point if any $none$

f) the relative maximum value $none$

g) the coordinates of the relative minimum point if any $(3, -6)$

h) the relative minimum value $y = -6 \; which \; occurs \; when \; x = 3$

15) $f(x) = x^2 - 3$

a) $f'(x) = 2x$

b) $\frac{2x}{2} = \frac{0}{2}$ $\quad x = 0$

C-h

$-\infty$ $\quad 0 \quad$ ∞

$(-\infty, 0)$ $\quad$ $(0, \infty)$

$x = -1$

$f'(-1) = 2(-1) = -2$

negative

Decreasing

$x = 1$

$f'(1) = 2(1) = 2$

positive

Increasing

y-coord min

$y = f(0) = 0^2 - 3 = -3$

min $(0, -3)$

a) f'(x) $f'(x) = 2x$

b) the critical numbers $x = 0$

c) interval(s) where the graph is increasing. $(0, \infty)$

d) interval(s) where the graph is decreasing. $(-\infty, 0)$

e) the coordinates of relative maximum point if any *none*

f) the relative maximum value *none*

g) the coordinates of the relative minimum point if any $(0, -3)$

h) the relative minimum value $y = -3$ *which occurs when* $x = 0$

17) $f(x) = x^3 - 12x + 4$

a) $f'(x) = 3x^2 - 12$

b) $3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$3(x+2)(x-2) = 0$

$3 = 0$ No Solution

$x + 2 = 0$, $x = -2$

$x - 2 = 0$, $x = 2$

$\boxed{x = \pm 2}$

C-h

$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$x = -3$	$x = 0$	$x = 3$
$f'(-3) = 3(-3)^2 - 12 = 15$	$f'(0) = 3(0)^2 - 12 = -12$	$f'(3) = 3(3)^2 - 12 = 15$
Positive Increasing	negative Decreasing	Positive Increasing

ycoord max $y = f(-2) = (-2)^3 - 12(-2) + 4$

$y = 20$

max $(-2, 20)$

ycoord min $y = f(2) = (2)^3 - 12(2) + 4$

$= -12$

min $(2, -12)$

a) f'(x) $f'(x) = 3x^2 - 12$

b) the critical numbers $x = 2, -2$

c) interval(s) where the graph is increasing. $(-\infty, -2) \cup (2, \infty)$

d) interval(s) where the graph is decreasing. $(-2, 2)$

e) the coordinates of relative maximum point if any $(-2, 20)$

f) the relative maximum value $y = 20$ *which occurs when* $x = -2$

g) the coordinates of the relative minimum point if any $(-2, -12)$

h) the relative minimum value $y = -12$ *which occurs when* $x = -2$

19) $f(x) = -x^3 - 3x^2 + 45x - 5$

ⓐ $f'(x) = -3x^2 - 6x + 45$

ⓑ $-3(x^2 + 2x - 15) = 0$

$-3(x+5)(x-3) = 0$

$-3 = 0$ No Solution

$x + 5 = 0$, $x = -5$

$x - 3 = 0$, $x = 3$

$\boxed{x = -5, 3}$

C-h

$-\infty$		-5		3		∞

$(-\infty, -5)$ $x = -6$

$f'(-6) = -3(-6)^2 - 6(-6) + 45$
$= -27$
Decreasing

$(-5, 3)$ $x = 0$

$f'(0) = -3(0)^2 - 6(0) + 45$
$= 45$
Increasing

$(3, \infty)$ $x = 4$

$f'(4) = -3(4)^2 - 6(4) + 45$
$= -27$
Decreasing

y-coord max $y = f(3) = -1(3)^3 - 3(3)^2 + 45(3) - 5$
$= 76$
max $(3, 76)$

ycoord min $y = f(-5)$,
$= -1(-5)^3 - 3(-5)^2 + 45(-5) - 5$
$= -180$
Min $(-5, -180)$

a) f'(x) $f'(x) = -3x^2 - 6x + 45$

b) the critical numbers $x = -5, 3$

c) interval(s) where the graph is increasing. $(-5,3)$

d) interval(s) where the graph is decreasing. $(-\infty, -5) \cup (3, \infty)$

e) the coordinates of relative maximum point if any $(3,76)$

f) the relative maximum value $y = 76$ *which occurs when* $x = 3$

g) the coordinates of the relative minimum point if any $(-5, -180)$

h) the relative minimum value $y = -180$ *which occurs when* $x = -5$

21) $f(x)=\frac{x+2}{x-5}$

(a) Denom $x-5$ num $x+2$

Deriv 1 Deriv 1

$$f'(x)=\frac{1(x-5)-1(x+2)}{(x-5)^2}$$

$$f'(x)=\frac{1x-5-1x-2}{(x-5)^2}$$

$$f'(x)=\frac{-7}{(x-5)^2}$$

(b) $-7=0$

No Solution

$(x-5)(x-5)=0$

$x=5$

$\boxed{x=5}$

c-h

$-\infty$ 5 ∞

$(-\infty,5)$

$x=0$

$f'(0)=\frac{-7}{(0-5)^2}=\frac{-7}{25}$

Decreasing

$(5,\infty)$

$x=6$

$f'(6)=\frac{-7}{(6-5)^2}=-7$

Decreasing

Always Decreasing

Never Increasing

no max / no min

a) f’(x) $f'(x)=\frac{-7}{(x-5)^2}$

b) the critical numbers $x=5$

c) interval(s) where the graph is increasing. never

d) interval(s) where the graph is decreasing. $(-\infty,5)\cup(5,\infty)$

e) the coordinates of relative maximum point if any none

f) the relative maximum value none

g) the coordinates of the relative minimum point if any none

h) the relative minimum value none

23) $f(x) = \frac{x-4}{x+1}$

(a) Denom $x+1$ Num $x-4$

Deriv 1 Deriv 1

$$f'(x) = \frac{1(x+1) - 1(x-4)}{(x+1)^2}$$

$$f'(x) = \frac{x+1-x+4}{(x+1)^2}$$

$$f'(x) = \frac{5}{(x+1)^2}$$

b) $5=0$
No Solution

$(x+1)(x+1)=0$
$x=-1$

C-h

$-\infty$ -1 ∞

$(-\infty, -1)$
$x=-2$
$f'(-2) = \frac{5}{(-2+1)^2} = 5$
Increasing

$(-1, \infty)$
$x=0$
$f'(0) = \frac{5}{(0+1)^2} = 5$
Increasing

Never decreases
no max
no min

a) f'(x) $f'(x) = \frac{5}{(x+1)^2}$

b) the critical numbers $x = -1$

c) interval(s) where the graph is increasing. ~~never~~ $(-\infty, -1) \cup (-1, \infty)$

d) interval(s) where the graph is decreasing. ~~$(-\infty, -1) \cup (-1, \infty)$~~ never

e) the coordinates of relative maximum point if any none

f) the relative maximum value none

g) the coordinates of the relative minimum point if any none

h) the relative minimum value none

25) $f(x) = xe^{3x}$

(a) 1st x | 2nd e^{3x}

Deriv 1 | Deriv $3e^{3x}$

$$f'(x) = x \cdot 3e^{3x} + 1e^{3x}$$

$$f'(x) = e^{3x}(3x+1)$$

(b) $e^{3x} = 0$

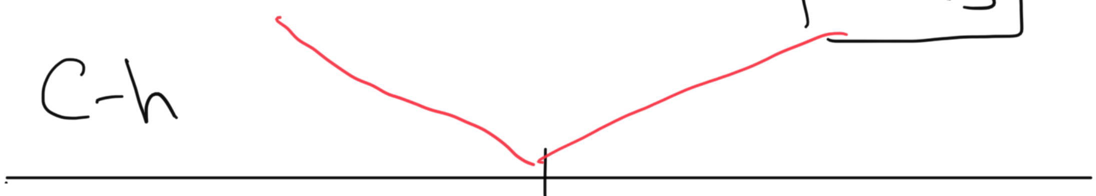
no Solution

$3x+1=0$

$3x=-1$

$\boxed{x=-1/3}$

C-h



$-\infty$ | $-1/3$ | ∞

$x=-1$

$f'(-1) = e^{3(-1)}(3(-1)+1)$

CAlculator -0.099

Decreasing

$x=0$

$f'(0) = e^{3(0)}(3(0)+1)$

$=1(1)$

$=1$

Increasing

no max

y-coord min $y = f(-1) = -1e^{3(-1)}$

$= -1e^{-3}$

$= -1/e^3$

min $(-1, -1/e^3)$

a) f′(x) $f'(x) = e^{3x}(3x+1)$

b) the critical numbers $x = -1/3$

c) interval(s) where the graph is increasing. $\left(-\frac{1}{3}, \infty\right)$

d) interval(s) where the graph is decreasing. $\left(-\infty, -\frac{1}{3}\right)$

e) the coordinates of relative maximum point if any none

f) the relative maximum value none

g) the coordinates of the relative minimum point if any $\left(-\frac{1}{3}, \frac{-1}{3e}\right)$

h) the relative minimum value $y = -\frac{1}{3e}$ *which occurs when* $x = -1/3$